Week 3 - Derivative, Tangent and Normal Lines

Instructions. Feel free to discuss with your group mates and do the following problems. You are not expected to finish all the problems. :)

Topic 1: Derivative Computing

Common Derivatives that we learned so far:

$(x^n)' =$ nx^{n-1} $\quad\quad$ $\frac{d}{dx}[e^x] =$ e^x

1. Compute the Derivatives of the following functions (No need to use definition.)

   (a) $y = 5x^3 + \sqrt{x}$

   (b) $y = e^x + 5x^{\frac{3}{5}} - 4x^{\frac{4}{7}}$

   (c) $f(x) = \frac{1}{x^2} + \sqrt[3]{x}$

   (d) $f(x) = \dfrac{x^3 - 2x^2 - 3x + 5}{x^2}$

   ~~(e)~~ $y = (3x^2 - \sqrt{x} + 1)^5$] "power rule" (OK to do it later when "power rule" is covered in more details)

   ~~(f)~~ $y = \sqrt{1 + x^3}$

(a) $y = 5x^3 + x^{\frac{1}{2}}$

$y' = 15x^2 + \frac{1}{2}x^{-\frac{1}{2}}$

(b) $y' = e^x + 3x^{-\frac{2}{5}} - \frac{16}{7}x^{-\frac{3}{7}}$

(c) $f(x) = x^{-2} + x^{\frac{1}{3}}$

$f'(x) = -2x^{-3} + \frac{1}{3}x^{-\frac{2}{3}}$

(d) $f(x) = x - 2 - 3x^{-1} + 5x^{-2}$

$f'(x) = 1 + 3x^{-2} - 10x^{-3}$

(e) $y' = 5(3x^2 - \sqrt{x} + 1)^4 (6x - \frac{1}{2}x^{-\frac{1}{2}})$

(f) $y = (1 + x^3)^{\frac{1}{2}}$

$y' = \frac{1}{2}(1 + x^3)(3x^2)$

Topic 2: Tangent line, Normal Line

2. Consider the parabola of $y = 4x - 4 + 3x^2$.

   (a) What is the equation of the tangent line of this parabola at (1, 3)?

   (b) What is the equation of the normal line of this parabola at (1, 3)?

   (c) At what point does the normal to this parabola at (1, 3) intersect the parabola a second time? *(Due to the time limitation, it's ok just to set up the equation and not solve it.)*

(a) $y' = 4 + 6x$

@ $x=1$, $y' = 4+6 = 10$

Tangent: $y-3 = 10(x-1)$

(b) Normal

slope $= -\frac{1}{10}$

Normal line:

$y-3 = -\frac{1}{10}(x-1)$

(c) $y = 4x-4+3x^2$

$y = -\frac{1}{10}(x-1)+3$

$4x-4+3x^2 = -\frac{1}{10}(x-1)+3$

$3x^2+4.1x-7.1=0$

$x = \frac{-4.1 \pm \sqrt{4.1^2+4\times 3\times 7.1}}{6}$

3. At which point on the curve $y = -\pi + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$?

$y' = 2e^x - 3$

slope of $3x-y=5$ is 3

$\Rightarrow \quad 2e^x - 3 = 3$

$e^x = 3 \qquad x = \ln 3$

4. For what value(s) of x does the graph of $f(x) = 4x^3 - 12x - 2017$ have a horizontal tangent?

$f'(x) = 12x^2 - 12 = 0$

$x = \pm 1$